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ON THE PROBLEM OF PHOTOGRAMMETRIC REFRACTION

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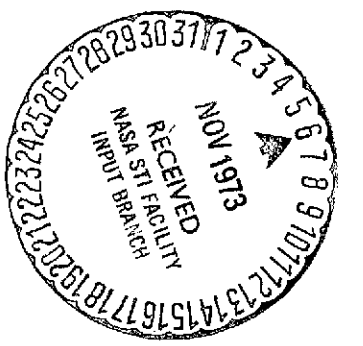

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16. Abstract

The present work is a continuation and the latest development of the work "On Photogramatic Refraction"(Bulletins of Higher Educational Institutions, "Geodezia i Aerofotos'emka," 1969, No. 1, Moscow); where general formulae used to obtain the angle of photogramatic refraction are derived through the total refraction A. For A, an expression was given, where two terms S_1 and S_2, are introduced, for the calculation of which it was necessary to know the relation for the distribution of the density of the atmosphere from the surface of the Earth to the point of photographing. In the present work, the density of the atmosphere is assumed, for the calculation of S_1 and S_2, to be standard (CIRA), and it is approximated regionally (cont)

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ON THE PROBLEM OF PHOTOGRAMMETRIC REFRACTION

S. V. Gromov

Article [1] gave formulas for determining photogrammetric refraction $\Delta\nu_2$ and geodesic refraction $\Delta\nu_1$; the major difficulty in determining these values lines in finding the value of complete refraction. /207*

$$A = -\int_{n_1}^{n_2} \operatorname{tg} i \frac{dn}{n},$$

where n is the refractive index of the air; i is the angle between the ray of light and the perpendicular line that is the radius of the spherical surface that divides a layer of air of one constant density from a layer of air of another constant density (a spherical model of the atmosphere was considered). Point 1 was situated on the Earth and point 2 was above the Earth. An expression for complete refraction was derived:

$$A = \operatorname{tg} i_1 (u_2 + u_2^2 - S_1 + S_2) + \operatorname{tg}^3 i_1 \left(\frac{u_2^2}{2} - S_1 + \frac{5}{2} S_2\right) + \operatorname{tg}^5 i_1 \left(\frac{3}{2} S_2\right) + \ldots \qquad (1.1)$$

In order to calculate all of the terms in (1) that do not contain S_1 and S_2 it is necessary to know only u at the end points of the part of the light ray under consideration, and i_1, the angle of incidence of the ray at point 1. In part 1 the law of the change of density of the air assumes only a spherical form

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*Numbers in right hand margin indicate pagination of foreign text.

of layers of a constant density; in part 2 it does not have any effect on the path of light here. But the terms S_1 and S_2 cannot be derived in part 1 and 2 without knowing the law of the distribution of the density of the atmosphere. Actually, in order to determine

$$S_1 = \int_1^2 s\,du \qquad S_2 = \int_1^2 s^2\,du$$

one must introduce some sort of single independent variable under the integral signs of S1 and S2. It is natural to use the density of the air δ for this purpose, using the dependence of δ on the height h of the point above sea level. It is known that the density of the air is related to the height above sea level by the following approximate function:

$$\delta(h) = \delta_0 q^h, \tag{1.2}$$

where δ_0 is the density of the air at sea level; h is the elevation in kilometers; q is some coefficient. Formula (1.2), which contains two parameters δ_0 and q = 0.9, can entirely satisfactorily give $\delta(h)$ for small elevations, but any further deviation of the actual density from this, which is obtained by formula (1.2), becomes unacceptably great. Therefore it is natural to divide the area where there is an atmosphere that can in any manner exert an influence on the refraction of light into a series of zones (spherical layers), and in each zone to present the density of the air in the form $\delta(h)$, where k(1,2, ..., m) is the number of the zone of division of the atmosphere. Now the density of the entire atmosphere is given through 2m parameters. If ρ is the distance from the center of the spherical Earth, then $h = \rho - \rho_0$ where ρ_0 is the radius of the Earth, and

$$\delta(\rho) = \delta_{0k} q_k^{(\rho-\rho_0)}. \tag{1.3}$$

The variable s, which appears in the expressions for S_1 and S_2,

is determined from the relation $1 + s = \frac{\rho}{\rho_1}$, where ρ_1 is the distance of point 1 from the Earth. But it is possible to note

$$1+s=\frac{\rho}{\rho_0+(\rho_1-\rho_0)}.$$

Let us designate $\rho_1-\rho_0$ by ε, where $\frac{\varepsilon}{\rho_0}$ is the small value of approximately 1/1000. Using the fact that $\frac{\varepsilon}{\rho_0}^2$ is a small value on the order of S^3, one can note that

$$s=\left(\frac{\rho-\rho_0}{\rho_0}\right)-\frac{\xi}{\rho_0}+o\left(s^{5/2}\right).$$

Using equation (1.3) and understanding that $du = \beta\frac{d\delta}{\delta_1}$, where $\beta = \frac{k\delta_1}{1+k\delta_1}$, we get

$$S_1=-\frac{\xi}{\rho_0}u_2+\frac{\beta}{\rho_0\delta_1}\sum_{k=1}^{m}\int_{A_k}^{B_k}\frac{\ln\delta-\ln\delta_{0k}}{\ln q_k}d\delta. \qquad (1.4)$$

In (1.4) A_k and B_k are the lower and upper points of the intersection of the ray of light with the borders of the k'th zone, while A_1 is coincident with point 1, and B_m - c is coincident with point 2 of the beam. After integration and simple rearrangements we derive S_1 in the form

$$S_1=-\frac{\xi}{\rho_0}u_2+\frac{\beta}{\rho_0\delta_1}\sum_{k=1}^{m}\left[s_k(A_k)-s_k(B_k)\right], \qquad (1.5)$$

$$s_k=\left(-\frac{\delta}{\ln q_k}\right)\left(1-\ln\frac{\delta}{\delta_{0k}}\right).$$

In order to determine S_2 it is sufficient to take s^2 in the form $s^2 = \frac{\rho-\rho_0}{\rho_0}^2$, and it follows that

$$S_2=\sum_{k=1}^{m}\int_{A_k}^{B_k}s^2du$$

or

$$S_2=\sum_{k=1}^{m}\left(-\frac{\beta}{\delta_1\rho_0^2}\right)\left[\frac{1}{\ln q_k}\right]^2\int_{A_k}^{B_k}\left(\ln\frac{\delta}{\delta_{0k}}\right)^2d\delta.$$

After integration by parts we get S_2 in the form

$$S_2 = \left(\frac{\beta}{\rho_0^2 \delta_1}\right) \sum_{k=1}^{m} [\sigma_k(A_k) - \sigma_k(B_k)], \tag{1.6}$$

where

$$\sigma_k = \frac{\delta_{0k}}{(\ln q_k)^2}\left[2\left(1 - \ln\frac{\delta}{\delta_{0k}}\right)\frac{\delta}{\delta_{0k}} + \frac{\delta}{\delta_{0k}}\left(\ln\frac{\delta}{\delta_{0k}}\right)^2\right].$$

Finding the numerical values of $\delta_k(A_k)$, $\delta_k(B_k)$, $s_k(A_k)$, $S_k(B_k)$ for some concrete model of the atmosphere will be the subsequent problem.

1. CALCULATING S_1 AND S_2 FOR A STANDARD ATMOSPHERE

In order to calculate S_1 and S_2 it is necessary to know the law of the distribution of the density in section 1.2 of the beam. For these goals we use the CIRA model of the atmosphere (Cospar International Reference Atmosphere); the numerical data is taken from the Geophysical handbook [2]. The values of the density that are assumed for the determination of S_1 and S_2 are taken at elevations of each even-numbered kilometer (from 0 to 90 km above sea level). The influence of the layer of the atmosphere that is situated from 87 to 800 km above sea level on the value A is less than 0".0005; therefore the atmosphere that is higher than 90 km is not taken into consideration when determining S_1 and S_2. For this, the angle i_1 should not be greater than 60 degrees.

The spherical layer of atmosphere that is included between the elevations of 0 - 90 km will be divided into six more precise layers:

the first	layer	is	between	elevations	. . .	0- 8 km,
second	"	"	"	"	. . .	8-18 km,
third	"	"	"	"	. . .	18-32 km,
fourth	"	"	"	"	. . .	32-50 km,
fifth	"	"	"	"	. . .	50-70 km,
sixth	"	"	"	"	. . .	70-90 km.

In layers 0-18 km, 18-32, 32-50 and 50-70 the density of the atmosphere at the lower boundary is approximately 10 times greater than the density at the upper boundary. We have divided layer 0-18 km even further into two sublayers since the densest mass of air is concentrated just here, and have obtained the subdivision that was cited earlier. In each of the six layers the density is approximated by the formula

$$\delta(h)=\delta_{0k}q_k^h. \tag{2.1}$$

Since an equation of the form (2.1) is difficult to solve with respect to the unknowns, δ_{0k}, q_k, then, assuming that we know the approximate values of δ_{0k} and q_k, we can note that

$$\delta_{0k}q_k^h=\bar{\delta}_{0k}\bar{q}_k^h+\Delta\delta_{0k}\bar{q}_k^h+\bar{\delta}_{0k}h\bar{q}_k^{(h-1)}\Delta q_k+\ldots$$

After this we come to an additional system of equations of the form

$$\Delta\delta_{0k}\bar{q}_k^h+\bar{\delta}_{0k}h\bar{q}_k^{(h-1)}\Delta q_k+l(h)=v. \tag{2.2}$$

This includes $\Delta\delta_{0k}$, Δq_k, which is the correction factor to the approximate value δ_{0k}, q_k, and v, the discrepancy of the equation, but

$$l(h)=\bar{\delta}_{0k}\bar{q}_k^h-\delta(h).$$

For each of the six layers equations of the form (2.2) have been formulated and solved by the method of least squares. These calculations were made by G. I. Govorov. The results are given in Table 1.

TABLE 1

Number of the zone	h, km	q_k	$10^3\delta_{ok}$, G/cm^3
1	0- 8	0.9039	1.222
2	8-18	0.8689	1.607
3	18-32	0.8547	2.009
4	32-50	0.8614	1.558
5	50-70	0.8860	0.421
6	70-90	0.8430	15.81

The approximate values of the values for the solution of system (2.2), pertaining to the first zone, were taken to be $q_1 = 0.9$ and $\delta_{01} = 1.23 \cdot 10^{-3}$. Then the precise values of the parameters for the k-1 zone were taken from the approximate values for the k'th zone.

The results of the calculation of S_k and of δ_k are placed in table 2. In the fourth and sixth columns the second number expresses in seconds of arc the influence of the indicated zone on S_1 and S_2; for the calculation it is assumed that

$$\delta_1 = 1, \text{ a } \rho_0 = \sqrt{MN}\big|_{B=45^\circ} = 6378{,}21 \text{ km}.$$

From the same table it is clear that it is possible to neglect the influence of zone No. 6 for the calculation of S_1, and that it is possible to not take into account zones No. 6 and 5 for the calculation of S_2. The presented table of values significantly facilitates the calculation of the terms S_1 and S_2.

TABLE 2

Zone Number	h(km	S_k	$S_k(A_k)-S_k(B_k)$	δ_k	$\delta_k(A_k)-\delta_k(B_k)$
1	0	9.35372	1.81659	185.14884	9.00471
	8	7.53713	0".0172	176.14413	0".00001
2	8	6.10290	3.61547	112.69488	45.20531
	18	2.48742	0".0342	67.48957	0".00007
3	18	2.24265	1.85044	58.38485	42.92168
	32	0.39221	0".0175	15.46317	0".00006
4	32	0.39367	0.35401	15.69200	13.41309
	50	0.03966	0".0033	2.27890	0".00002
5	50	0.04562	0.04014	2.71184	2.27833
	70	0.00548	0".0004	0.43351	0".00000
6	70	0.00931	0.00931	0.70840	0.70840
	90	0.0000	0".0001	0.00000	0".00000

2. CALCULATING THE VALUE OF THE ANGLE $\Delta\nu_2$ OF PHOTOGRAMMETRIC REFRACTION

When making a photographic survey from a height we usually know the angle i_1; therefore we will discuss the method of simultaneous calculation of the complete refraction and of photogrammetric refraction.

Let us assume that we know the approximate geodesic coordinates of the camera at the moment that we take the picture (B_2, L_2, H_2, as well as of the point being photographed which interests us (B_1, L_1, H_1). At the time of the survey we should know the meterological conditions at points 2 and 1. During global surveys the points of locality (type 1) can be assumed to be far from each other, although they are depicted in the same survey; therefore at each of these points the conditions of the state of the atmosphere can be different. Using the geodesic coordinates of points 1 and 2 we find their rectilinear coordinates x_i, Y_i, Z_i, and through the latter coordinates we determine the approximate length of the chord 1.2. We assume the Earth is a sphere of radius R equal to the average radius of curvature of the reference-ellipsoid under the latitude of point 2, we solve the triangle 012, where 0 is the center of the terrestrial sphere, and we find the angles v_2 and v_1. Since we know the angle i_1, then in the first approximation we assume that i_1 and v_1 are together; according to formula (1.1), using table 2, we calculate the approximate value of complete refraction A. Later, according to formula (3.1)

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$$\left.\begin{aligned}\Delta\nu_2 &= \frac{\varkappa''}{(n_1q_1 - n_2q_2)}\left[n_1q_1\left(A + \frac{P}{2q_1}\Delta\nu_1^2 - \frac{P}{2q_2}\Delta\nu_2^2\right) + P(n_2 - n_1)\right],\\ \Delta\nu_1 &= \frac{\varkappa''}{(n_1q_1 - n_2q_2)}\left[n_2q_2\left(A + \frac{P}{2q_1}\Delta\nu_1^2 - \frac{P}{2q_2}\Delta\nu_2^2\right) + P(n_2 - n_1)\right],\end{aligned}\right\} \qquad (3.1)$$

assuming in their right parts that $\Delta\nu_1 = \Delta_\nu 2\ 0$, we find $\Delta\nu_2$ and $\Delta\nu_1$ in the first approximation. Formulas (3.1) are quite different from the corresponding formulas given in our work [1], since in (3.1) in the first parentheses the terms $\frac{P}{2q_1}\Delta\nu_1^2$ and $\frac{P}{2q_2}\Delta\nu_2^2$ appear which were not there before. Detailed consideration of the problem indicated that these terms must be taken into account, since their influence on $\Delta\nu_1$ and $\Delta\nu_2$ can constitute a few seconds. In (3.1)

$$q_1=(R+H_1)\cos\nu_1,\quad q_2=(R+H_2)\cos\nu_2,$$
$$P=(R+H_1)\sin\nu_1=(R+H_2)\sin\nu_2.$$

In the second approximation ($i_1 = \nu_1 + \Delta\nu_1$). With this i_1 we again calculate A, and later according to formula (3.1) we find $\Delta\nu_2$ and $\Delta\nu_1$ secondarily, having substituted into (3.1) the $\Delta\nu_1$ and $\Delta\nu_2$ that were found in the first approximation. The process of approximations is completed when A stops changing within a given precision for the calculation of A. But the equality $\Delta\nu_2 - \Delta\nu_1 = A$ will be achieved only with precision of the calculation of $\Delta\nu_1$ and $\Delta\nu_2$ (which is usually less than the precision of the calculation of A). Below we present an example of the calculation of the angle of photogrammetric refraction; per unit density of the air we have assumed the density of dry air for a temperature of 0 degrees Centigrade and a pressure of 700 mm Hg, which equals $1.2932 \cdot 10^{-3}$ e/cm^3. In the formula $n = 1+k\delta$ we have taken $k = 0.00029234$ [3]. In our example we assumed $H_1 = 0$, $H_2 = 50.0$ km, $\nu_1 = 60$ degrees, $\delta_1 = 1.0000000$, /213
$\delta_2 = 0.0006959$, $R = 6378.2$ km; we derived $n_1 = 1.0002923$, $n_2 = 1.0000002$, $\beta = 0.00029226$, $u_2 = 0.00029206$, $q_1 = 3189.1$ km, $q_2 = 3287.3$ km, $P = 5523.5$ km.

The numerical data of the process of approximation is cited in table 3.

TABLE 3

Approximation	l_1	S_1	S_2	A"
1-e	60°00'00"	0".0722	0".0002	103".923
2-e	59 58 30	--	--	103.818
3-e	59 58 32	--	--	103.820
4-e	59 58 32	--	--	103.820

Approximation	A	$\Delta\nu_2$	$\Delta\nu_1$	$\Delta\nu_2-\Delta\nu_1$
1-e	0.00050383	13".1	-90".8	103".9
2-e	0.00050332	15.5	-88.4	103.9
3-e	0.00050333	15.4	-88.5	103.9
4-e	0.00050333	15.4	-88.5	103.9

The precision of determining $\Delta\nu_2$ according to formula (3.1) depends on the precision with which the important terms A and P which appeared in (3.1) were determined. For various values ν_1 and various elevations B_2 the coefficients in (3.1) for A will have various values. If we take ν_1 = 60 degrees, then we obtain a table of approximate values of the coefficient

$$Q=\frac{n_1q_1}{n_2q_2-n_1q_1}.$$

TABLE 4

H, km	Q	m_A''	m, p km	$m_{q\nu}m_{q2}$, km
25	60	0.001	0.08	0.05
50	30	0.003	0.16	0.1
100	15	0.01	0.3	0.2
250	5	0.02	0.8	0.5
1000	2	0.05	3	2

In the second column of the table we have indicated the absolute values of the allowable errors for A for which the absolute error in $\Delta\nu_2$ will be less than 0".1. In the third column we gave the allowable errors in P with respect to the errors in $\Delta\nu_2$ under the same assumptions. The precision of the values of q_1 and q_2 should not be lower than the limits indicated in the fourth column of the table.

CONCLUSION

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Determining the figure of the Earth with the help of a precise photographic survey from a great height is a real problem of modern geodesy. Over the past ten years a series of interesting works have come to light which bear on the precise estimation of the influence of the atmosphere during high photographic mapping.

The given article is for the purpose of assisting in the solution of this problem.

16. Abstract (continuation) by the formula $\delta = \delta_0 q_k^h$. The components S_1 and S_2 are given for each region; the method of computation of A is shown, when i_1 (the inclination of the ray with respect to the vertical) does not change. The method of calculation of the photogramatic refraction is completed up to the practical use for high altitude surveys.

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